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A NEW ELECTRONIC MULTIPLICATION
METHOD INVOLVING ONLY SIMPLE
CONVENTIONAL CIRCUITS

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A NEW ELECTRONIC MULTIPLICATION METHOD
INVOLVING ONLY SIMPLE CONVENTIONAL CIRCUITS

TABLE OF CONTENTS

	<u>Page</u>
I. INTRODUCTION	1
II. DISCUSSION OF THE METHOD	3
III. THE CIRCUIT	8
IV. TEST RESULTS	12
V. DISCUSSION AND CONCLUSIONS	15
APPENDIX	17
Table of Contents	18

ABSTRACT

If e_1 and e_2 represent two complex time functions and e_c is a large high frequency carrier, the average value of $|e_c + e_1 + e_2| - |e_c + e_1 - e_2|$ is equivalent to the product of e_1 and e_2 . When the functions are voltages this difference in magnitudes can be found with simple conventional circuits. This implies that electronic multiplication is not as difficult as other methods might lead one to conclude.

A circuit and the operating data on it are reported here to verify this implication. This new method is examined in sufficient detail to make clear why it works. Its suitability for use in a wide variety of multipliers is pointed out but it is presented chiefly as a solution to the problem of constructing multipliers for field use where simplicity, low cost, ease of maintenance, and ruggedness are features that have to be combined with accuracy and wide dynamic range.

A NEW ELECTRONIC MULTIPLICATION METHOD
INVOLVING ONLY SIMPLE CONVENTIONAL CIRCUITS

I. INTRODUCTION

It is common practice to add, subtract, integrate, or differentiate raw field data before recording it, but received signals are rarely, if ever, multiplied together in the field. The reasons for not multiplying may have been that (1) electronic multipliers were expensive and difficult to build and (2) the advantages of multiplying complex signals were not thought to justify the effort of developing instruments for doing it.

Recent developments have invalidated this second reason. The advantages of electronic computers which involve multiplication, particularly those involving correlation techniques for detecting small signals in the presence of noise, have been demonstrated theoretically and in laboratory experiments and several practical electronic multipliers have been reported.

As far as field use of multipliers is concerned, though, the first objection still applied until the development reported here was completed. Most other multipliers have excellent operating characteristics but they lacked the simplicity and freedom from delicate parts and adjustments that are vital for field equipment. The instrument described here is one that has these additional characteristics and appears to be well suited for field use. It is inexpensive, small, easy to construct and repair; its operation is stable and accurate over a wide dynamic range; and standard components can be used throughout the circuit.

These characteristics stem from using a new method of finding products. It is similar to previous methods in that actual multiplication is circumvented by using operations other than multiplication to produce a quantity that is equivalent to the desired product. In this method, however, a different quantity is produced. It is one that can be obtained from the inputs with simple conventional circuits. Thus the need for special parts and complex circuits which exists in most other multipliers is eliminated.

This presentation includes (1) a list of the steps to produce the new quantity, (2) an explanation of why it is proportional to the product of the inputs, (3) a circuit in which this method of finding products was used, (4) operating data on the circuit, and (5) a discussion on the amplitude and frequency limitations of the signals applied.

The method is suitable for wider use than just in field type instruments and deserves consideration for use in multipliers for all applications.

II. DISCUSSION OF THE METHOD

The object of the new method is to find the average value of the difference in magnitudes of two special signals produced from a combination of a high frequency sawtooth carrier and the inputs. It can be identified as the "difference in magnitudes" method.

To find the product of two complex time functions, e_1 and e_2 , by the difference in magnitudes method these steps need to be taken:

1. Add a large, high frequency, sawtooth carrier, e_c , to either e_1 or e_2 (use e_1 , for example) to obtain e_c+e_1 .
2. Add and subtract the other input signal, e_2 , to this sum obtaining $e_c+e_1+e_2$ and $e_c+e_1-e_2$.
3. Find the absolute magnitude of these quantities, $|e_c+e_1+e_2|$ and $|e_c+e_1-e_2|$.
4. Subtract the second of these absolute magnitudes from the first to obtain the difference $|e_c+e_1+e_2|-|e_c+e_1-e_2|$.
5. Average this difference. The result is $k e_1 e_2$ where k is a simple proportionality constant.

As long as the difference of step 4 is obtained, the order of the previous steps and the circuits used for taking them are unimportant. Of course, the accuracy of the method depends on the accuracy of the difference so high quality circuits should be used. The ease of satisfying this condition accounts for the advantages of the method.

To show that this procedure produces the product of e_1 and e_2 , the average of the difference, $\frac{1}{2}(|e_c+e_1+e_2|-|e_c+e_1-e_2|)$, must be shown to be equivalent to the product. To do this the waveform of the difference function will be examined.

At any instant the value of the difference is proportional to whichever of the two signals, $e_c + e_1$ or e_2 , is smaller. It is positive when both of the signals have the same sign but becomes negative when one signal is positive and the other is negative. These facts can be verified by inspection. First simplify the expression by letting $e_c + e_1$ equal a single signal, S_1 , and letting e_2 equal another signal S_2 . Then evaluate the expression with all possible relations between the amplitudes and signs of the S_1 and S_2 signals.

When e_c is a large high frequency sawtooth wave and e_1 and e_2 are sine waves, the difference function that satisfies the specified conditions is shown in Figures 1 and 2. In Figure 1, e_1 is zero; in Figure 2, e_1 is identical to e_2 . S_1 (equal to $e_c + e_1$) and S_2 (equal to e_2) are labeled in the figures. The heavy solid curve designated as e_0 is the difference function. This curve differs only slightly from what remains of the S_1 signal after the positive and negative peaks are clipped off at a value depending on the S_2 signal. The difference is that a reflection of the remainder about the zero axis is obtained when S_2 is negative. The reflection occurs because the difference is positive when both S_1 and S_2 are negative. The clipping occurs because the difference function follows the S_2 signal when that one is smaller but follows the S_1 signal when it becomes the smaller one. A conclusion that can be made immediately is that S_2 independently controls the heights of both the positive and negative pulses that appear. It follows from this as well as from the expression for the difference that the difference in-

magnitudes is zero when e_2 is zero. The effects of e_1 on the difference function will now have to be found.

The difference between Figure 1 and Figure 2 are due to e_1 . Its effects can be visualized by imagining the difference function to be the part of a plotted curve of the carrier that could be seen through a window in an opaque shield placed over the plot. This transparent section would be in the shape of the magnitude of the e_2 signal. When e_1 is zero the carrier axis would be right under the center axis of the window. A curve such as that labeled e_0 in Figure 1 would be seen. When e_1 is added the opaque shield would not be moved but the carrier plot would be shifted. It would move up at the points representing times when e_1 is positive and down when e_1 is negative. A curve like e_0 of Figure 2 would be seen through the window. Moving the carrier up and down this way would look through the window like e_1 to shift the slanting parts of the carrier sideways. This causes widening of some pulses and narrowing of the adjacent ones which have the opposite sign. The conclusion to be drawn from this is that e_1 controls the difference between the widths of the positive and negative pulses.

Since the vertical shift is proportional to e_1 and the carrier is linear, the changes in pulse width are proportional to e_1 . Figure 3 can be used to show that when this condition is satisfied and the pulse heights depend on e_2 the average value of the difference function is proportional to $e_1 e_2$.

Figure 3 is an enlarged drawing of some typical pulses. The dotted curve is the shape the difference function would have if e_1 were

zero; the solid curve is obtained when it is not. The shaded areas are the differences between positive and negative pulse areas due to e_1 . These areas are enclosed by parallelograms so they equal the product of the length of one pair of sides and the distance between them. The length of the sides in the direction of the zero axis is proportional to e_1 . The pulse heights and hence the distance between the sides of the parallelogram are proportional to e_2 . Therefore, the differences in positive and negative pulse areas per cycle of the carrier and hence the average value of the difference function is proportional to $e_1 e_2$.

In summary, a simple method of finding the waveform of the difference, $|e_c + e_1 + e_2| - |e_c + e_1 - e_1|$ was presented. The peculiar dependence of this wave on e_1 and e_2 when e_c is a sawtooth wave was pointed out. Then the average value of expression was shown to be proportional to e_1 times e_2 . This shows that the steps, listed in the opening paragraph produce the equivalent of multiplication.

From this explanation it can be seen that the peak carrier amplitude must be larger than the sum of the e_1 and e_2 amplitudes. If this condition did not exist, e_2 would not be smaller than $e_c + e_1$ when e_1 and e_c have opposite signs and it would not control the pulse heights. Making the carrier very large to make doubly sure it is large enough is not a good idea, though, because the multiplied output voltage for fixed inputs gets smaller when the carrier is increased. Referring to Figure 3, the slanting sides of the pulses get steeper when the carrier gets bigger so the horizontal shifting of these sides

would be less when e_1 shifted the carrier vertically. As a result the area changes due to e_1 would be smaller and there would be less multiplied output. As a compromise on the carrier size it can be set to equal the sum of the expected peak values of the e_1 and e_2 signals. Monitoring e_1 and e_2 is then advisable to make sure they do not exceed their expected values.

The carrier frequency should be as high as practical to get operation as discussed. It has no effect on the output voltage, but determines to some extent the complexity of the output averaging circuit. The higher the carrier frequency the simpler this circuit can be.

The explanation of how the method works also illustrates the fact that any wave that varies linearly with time would be an acceptable carrier. Selection of the one used can depend entirely on convenience of generating it and the necessary complexity of the circuits for using it.

For this discussion e_1 and e_2 were assumed to be sine waves so changes from normal shape would be easily recognized in the figures. The same conclusions would have been reached if complex inputs had been assumed, so the discussion should not be construed to indicate that the difference in magnitude method can only be used on sine waves.

III. THE CIRCUIT

The circuit presented illustrates the practicality and supports the validity of the foregoing procedure and analysis, but the main reason for presenting it is to illustrate the simplicity of the circuits required. This seems to justify including the detail schematic diagram of Figure 5.

Omitted from this diagram are a twin triode Potter type sawtooth wave generating circuit and the power supply which make it a complete multiplier. The two channels in the instrument, one for handling the S_1 signal and the other for S_2 , are labeled. The only difference between them up to the secondaries of the output transformers is a triode adding circuit for adding e_1 to e_c (step 1 in the procedure).

The amplifying, phase inverting, and power amplifying stages in each channel were included so that maximum multiplied output could be obtained with low level input signals. These circuits were also necessary for presenting signals to the output transformers so that all other steps of the procedure can be performed in the transformer secondary and following circuits.

The transformer secondaries were connected so that $(S_1 + S_2)$ and $(S_1 - S_2)$ (the quantities specified in step 2) would be produced in push pull relative to ground. The points at which the signals are obtained are labeled on the circuit diagram.

Step 3 (full wave rectification of the step 2 signals) is performed by the four diodes. The polarities of two of them are reversed

to transform the operation of subtracting required in step 4 to one of adding. This was done so that the difference effectively obtained would be represented by a voltage relative to ground. An enlarged drawing of the diode circuit is shown in Figure 4. The full wave rectified value of $S_1 + S_2$ appears at the point marked "A" in the circuit and the negative of this value of $S_1 - S_2$ appears at "B".

Step 4 is taken in the T-shaped resistor network between the diodes. The voltage at point "C" is the sum of the voltages at "A" and "B". This is where the difference in magnitudes appear.

The low pass RC filter connected at point "C" averages the difference in amplitudes to complete the procedure. The filter output is proportional to the product $e_1 e_2$.

Our specific use for this multiplier influenced the design of the circuit somewhat. A discussion of this use may answer questions on permissible differences between these and circuits in instruments for other applications.

In the first place a single ended output was desired. This may not always be true. If an ungrounded output is satisfactory, the one pair of diodes need not be reversed and the resistance T network in which some loss in signal is suffered need not be used. Also, both the $S_1 + S_2$ and the $S_1 - S_2$ signals would not have to be balanced to ground.

Transformer coupling to the diodes was possible because dc components in the inputs were not to be multiplied. Other coupling schemes would make multiplying dc signals possible and could be used

for ac signals too. Transformers were used in this case to get as large a signal as possible applied to the diodes without overdriving any of the previous stages and within the limitations of the 250 volt power supply used. This was desired because the maximum value of the output product and hence the dynamic range depends on how large a voltage can be applied to the diodes.

In this instrument a 5 kc sawtooth carrier was used and it was added at the input to the S_1 channel. The whole channel had to pass it without distortion so the channel frequency characteristic had to be flat up to the neighborhood of 50 kc. This explains the use of degenerative feedback around the voltage and power amplifying stages. Similar feedback was included in the S_2 channel to make the two channels enough alike that e_1 and e_2 signals at the diodes would have the same relative phases and amplitudes that they had back at the input terminals. Addition of the carrier at a later point in the circuit would relax the restrictions on the design of the earlier stages and reduce the need for feedback.

Potentiometers were included in the transformer secondary circuits to improve the balance and to compensate for slight differences in diode characteristics. Proper adjustment was found by observing the difference signal at point "C" in the circuit while e_1 , e_2 , and e_0 were applied separately. Settings for best operation did not have to be changed during several months of operation except when a diode was replaced.

The potentiometers in the S_1 channel input adding circuit were included for adjusting the relative amplitudes of the e_1 and e_c signals. Channel gain controls were included to prevent overdriving for large input signals but to permit increasing the gain to get plenty of output for small input signals. These controls effectively set the proportionality constant between the output voltage and the product of the inputs.

IV. TEST RESULTS

The operating characteristics of the multiplier were measured as sinusoidal input signals were applied. To check on the proportionality of the output to each input, the rms value of the inputs and the dc output were measured as one input was held constant and the other varied. Plots of the data obtained in this manner are presented in Figures 6 and 7. As these data were being obtained the peak to peak amplitude of the constant input was set at one-half the peak to peak amplitude of the carrier. Maximum allowable value for the other input signal was defined as the value at which the sum of the inputs equaled the carrier. The data shows that the output is proportional to both inputs within better than 1% below the maximum value of the inputs and that only slight deviations from proportionality occur when this value is exceeded.

A further test of multiplication accuracy was conducted by finding the amplitude of one input required to produce a given dc output for each of several values of the other input. If the output is the product of the two inputs, there should be a reciprocal relationship between the inputs for constant output so a log-log plot of one input versus the other should be a straight line. The data showing the relationship between the inputs are presented in Figure 8. Maximum deviation of the circled data points from the lines occurred for low values of output voltage. Lack of accuracy in measuring low values of both input and output voltages probably accounts for these deviations.

The dynamic range of the multiplier was defined as the ratio of the output obtained with maximum allowable inputs to the value obtained when either or both inputs were zero. When the inputs were zero, some circuit instabilities caused random drifts in the output that were barely detectable with a meter on which 10 millivolts can be read, so 10 millivolts was assumed to be the minimum attainable output. The dynamic range was calculated to be 500 to 1 on the basis of this zero drift and a 5 volt maximum output.

An interesting feature of this type of multiplier is that the accuracy and dynamic range do not depend on the dynamic characteristics of the diodes. Examination of what happens to the diodes when the circuit is multiplying shows that the carrier effectively just switches them on and off. They are either conducting because a large positive voltage is applied or non-conducting because of the presence of a large negative one. As a result the diodes only need to have an impedance that is low compared to the rest of the circuit when they are conducting and high when they are not conducting. This eliminates an objectionable necessity for using matched or specially selected diodes.

The average value of the product of two sine waves is proportional to the cosine of the phase angle between the waves. Data to demonstrate that this characteristic of the product is produced by this multiplier was obtained by reading the output voltages as the input amplitudes were held constant but the phase angle between the

Input sine waves was varied from zero to 180 degrees. A plot of the output versus the cosines of the phase angle is presented in Figure 9. If the phase angle between the inputs is considered to be a measure of the correlation between the two signals, this curve indicates the operation of the multiplier as a correlator and the conclusion can be reached that the output due to unrelated signals would be zero. This conclusion was verified by using unrelated noise signals and different frequency sine waves as inputs.

The foregoing data were obtained while the input sine waves were between 20 and 500 cycles per second and the time constant in the output filter was set between 1 second and 8 seconds. Neither the input frequency nor the filter time constant in these ranges of values had a measurable effect on the dc output voltages recorded.

To study instantaneous products the upper cutoff frequency of the output filter was raised to between 1 and 5 kc. As anticipated the ac output was at twice the input frequency. It varied linearly with the input amplitudes and was constant with frequency when the output frequency was below the value at which the attenuation of the output filter could be noted. With pure sine wave inputs the output should have had no components other than those at double the input frequency. However, a small input frequency component was detected in the output. By using extra care in adjusting the balance controls and improving the linearity of the sawtooth carrier, the amplitude of this component was reduced to less than one one-hundredth of the amplitude of the double frequency component.

V. DISCUSSION AND CONCLUSIONS

Rigor in justifying the use of the difference in amplitude method was sacrificed in this presentation in favor of brevity. Empirical results presented actually seem to be sufficient justification. The discussion on the waveform of the difference function was included mainly in an effort to give a picture of what occurs in the instrument as signal amplitudes vary. A better understanding can be obtained by observing the changes in the difference function wave shape on an oscilloscope. Getting curves quite similar to the ideal ones of Figures 1 and 2 is not difficult. In fact, this is a good way to evaluate the operation of a circuit.

In accomplishing the objectives of the difference in amplitude method the objectives of both the familiar "quarter-squares" and the pulse area methods are simultaneously accomplished. The brief analytical justification of the new method was based on the fact that it is a practical pulse area method. But the average of the magnitudes subtracted in the method can be shown to be the squares of the sum and difference of the inputs, so the objectives of the quarter-squares method are accomplished too. Since this is the case one might conclude that multipliers in which only one of the earlier methods is used were partially using the difference in amplitude method. Then the fact that using it completely requires simpler circuits also becomes interesting.

It has been suggested previously that diode circuits other than the one presented could be used. References for these other cir-

cuits would include discussions on phase discriminators and suppressed carrier modulators. The similarity between the diode circuits in these devices and the ones required for multipliers of the new type is marked. In fact with the carrier reduced to zero and one of the inputs kept much larger than the other, the multiplier presented in this paper becomes an excellent phase discriminator. With normal carrier amplitude but the signal designated as e_1 reduced to zero and no output filtering the multiplier becomes a suppressed carrier modulator. These similarities suggest the probability that many phase discriminators and suppressed carrier modulators can be converted into multipliers with only slight modifications.

While the operation of the circuit presented is of high caliber, the circuit was not intended to represent the ultimate in either size or precision that can be built to operate according to the new method. This particular circuit has frequency limitations which would not make it especially desirable for obtaining instantaneous products or for multiplying dc voltages. These are not limitations of the method however and could be overcome by designing the circuits for these and other applications.

It is hoped that this presentation sufficiently demonstrates the advantages of the difference in amplitude method that others who need electronic multipliers will realize useful assistance from it. Multipliers constructed to operate this way appear to be solutions to the problems of obtaining inexpensive, though accurate, multipliers for laboratory use and small, rugged instruments for field use.

This development was part of the work on a classified contract between the Office of Naval Research and the Magnolia Petroleum Company.

APPENDIX

APPENDIX

TABLE OF CONTENTS

	<u>Page</u>
Figure 1 - Output Voltage (With $e_1=0$)	A-1
Figure 2 - Output Voltage (With $e_1 = \text{Max. Value}$; $e_2 = \text{Max. Value}$)	A-2
Figure 3 - Typical Pulses When $e_1 = 0$ (dashed lines) and When $e_1 = e_2$ (solid lines)	A-3
Figure 4 - Circuit for Obtaining $ S_1+S_2 - S_1-S_2 $	A-4
Figure 5 - Schematic Diagram of Complete Multiplier	A-5
Figure 6 - Curve of Multiplier Output vs e_1 Input with e_2 at Limit Value	A-6
Figure 7 - Curve of Multiplier Output vs e_2 Input with e_1 at Limit Value	A-7
Figure 8 - e_2 Input Necessary to Produce Various Values of Output vs e_1 Input	A-8
Figure 9 - Multiplier Output vs Input Product with Varying Phase Angle	A-9

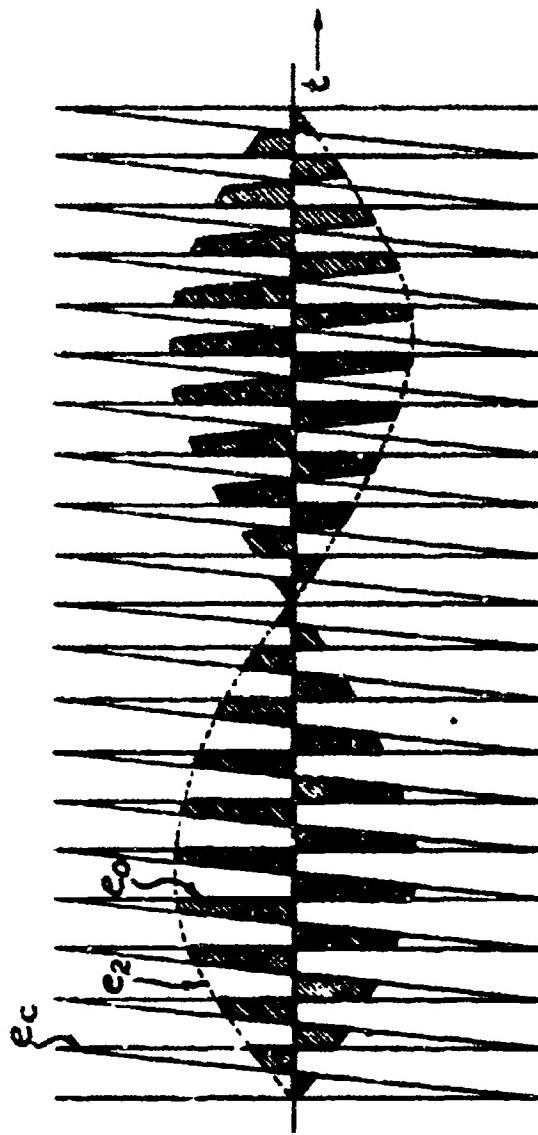
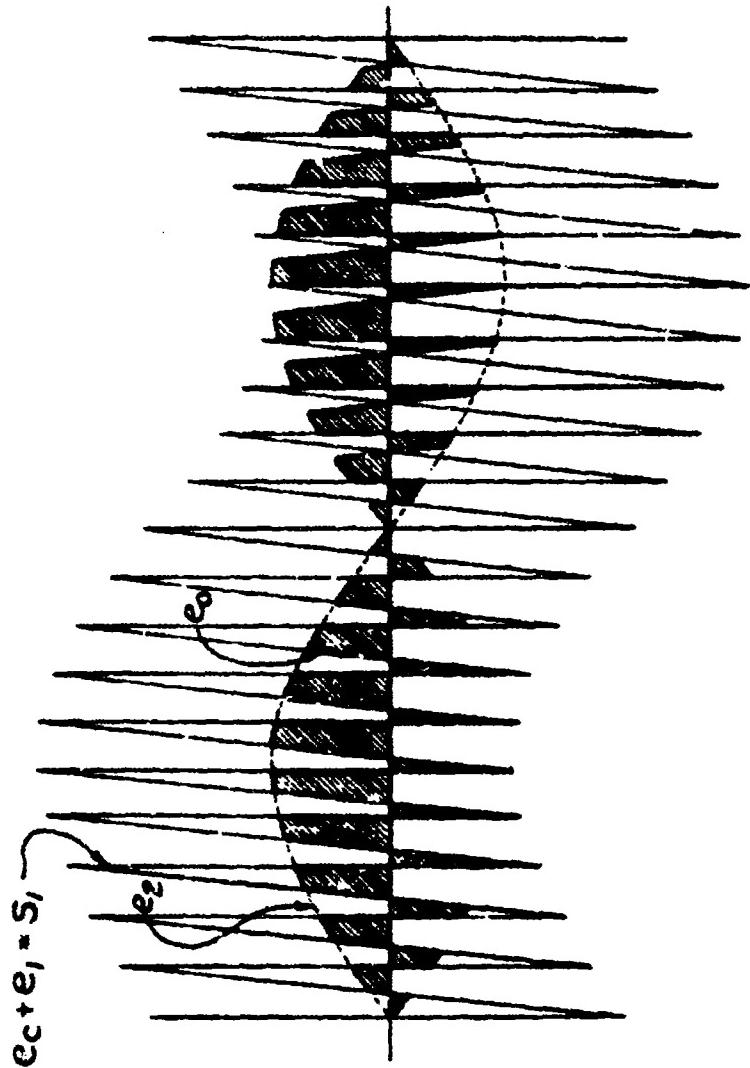


FIGURE !
Output VOLTAGE (With $e_i=0$)

FIGURE 2
OUTPUT VOLTAGE (WITH $e_1 = \text{Max. Value}; e_2 = \text{Max. Value}$)



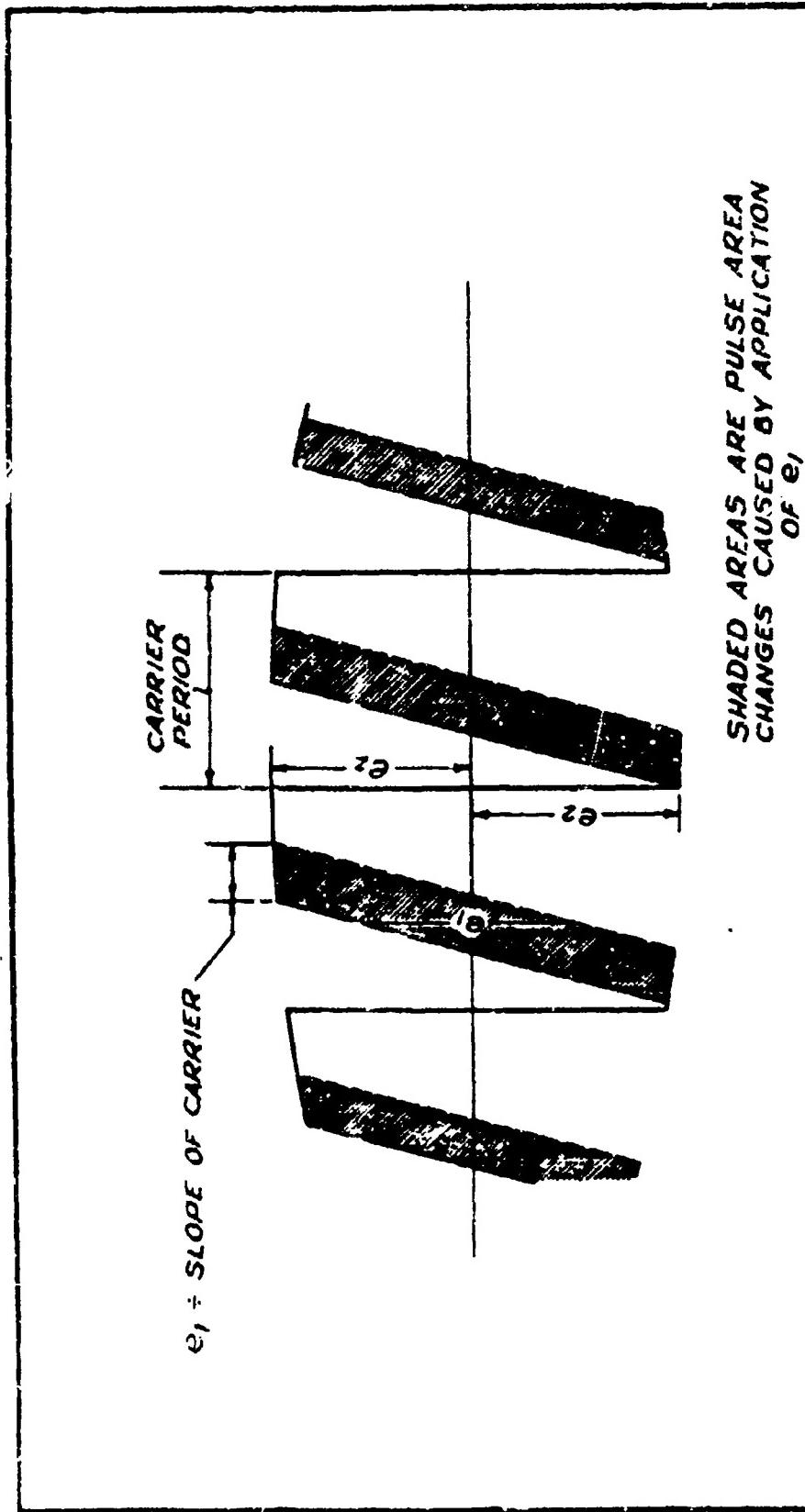


FIGURE 3
Typical Pulses
WHEN $e_1=0$ (Dashed Lines)
AND WHEN $e_1=e_2$ (Solid Lines)

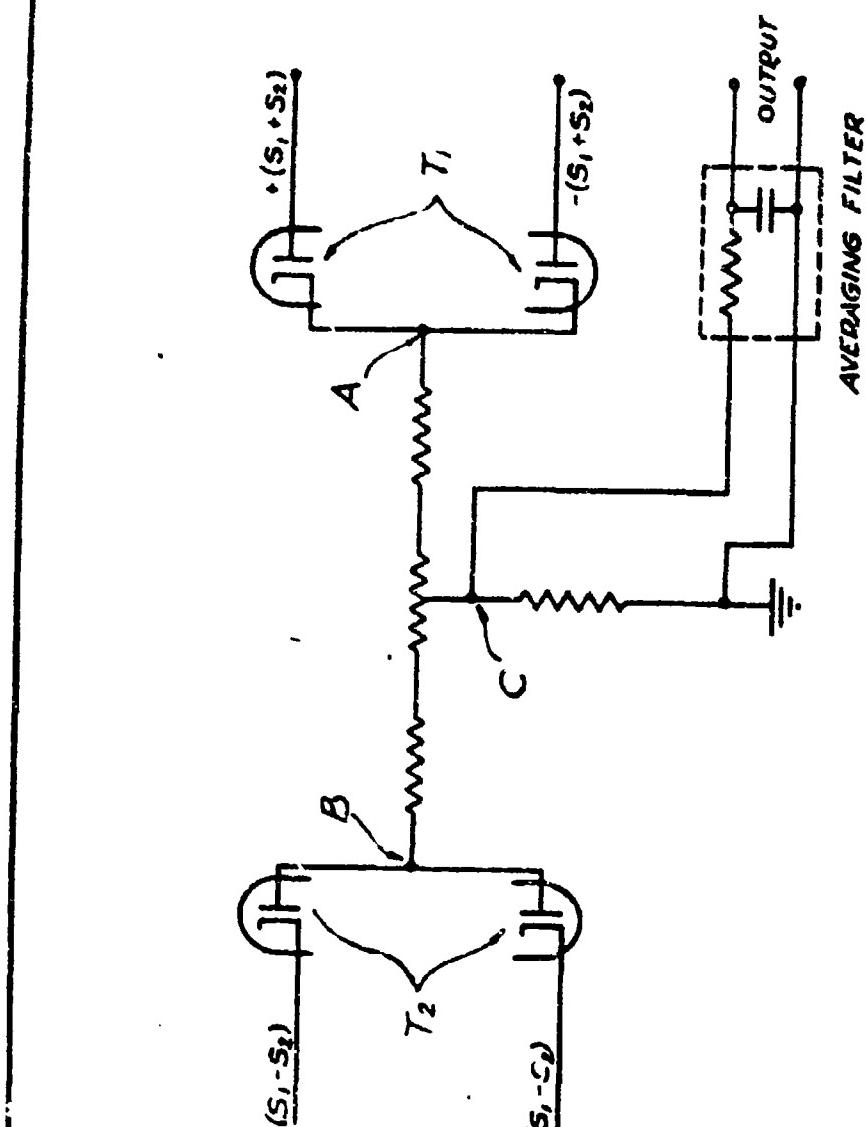


FIGURE 4
CIRCUIT FOR OBTAINING
 $\frac{S_1 + S_2}{|S_1 - S_2|}$

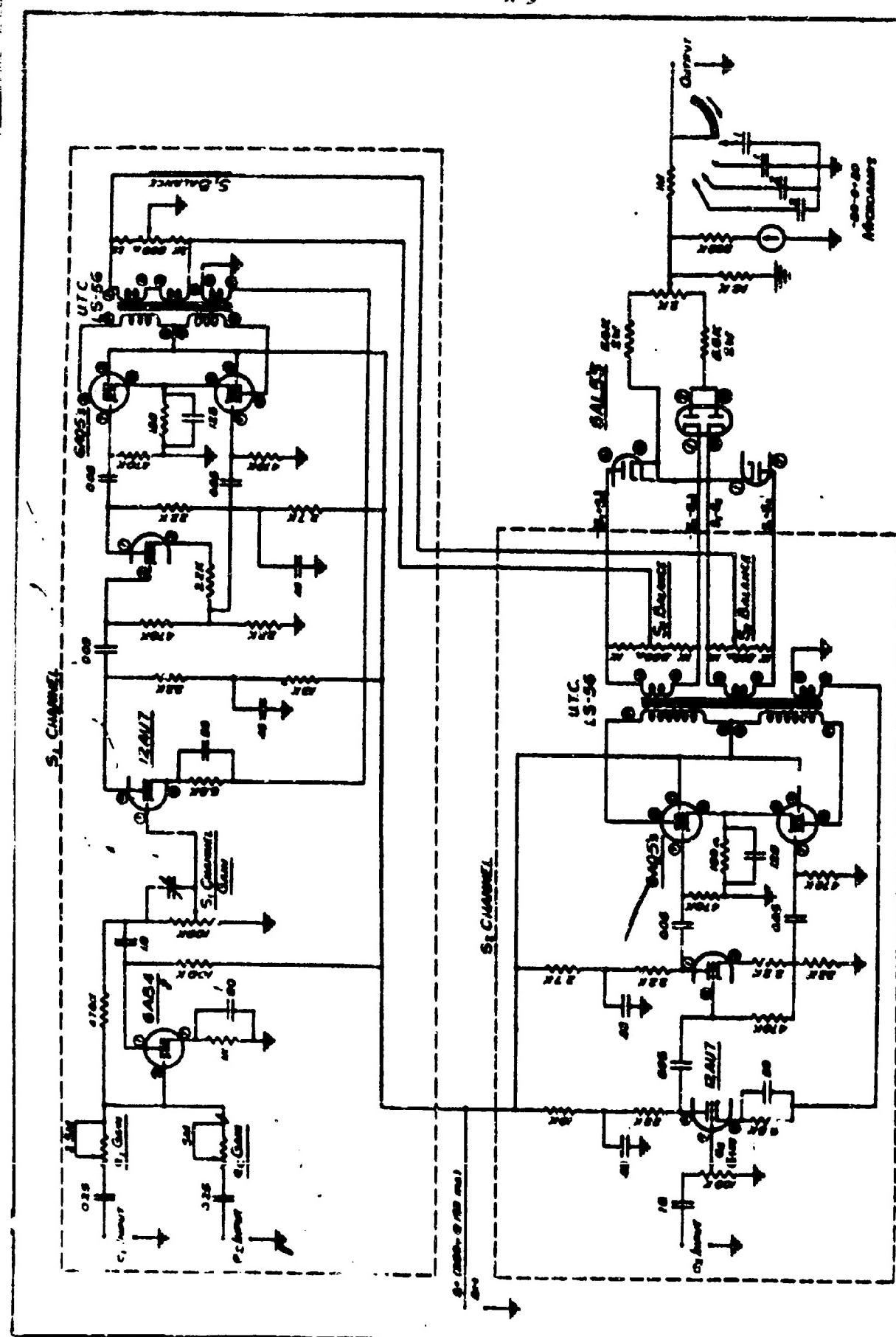
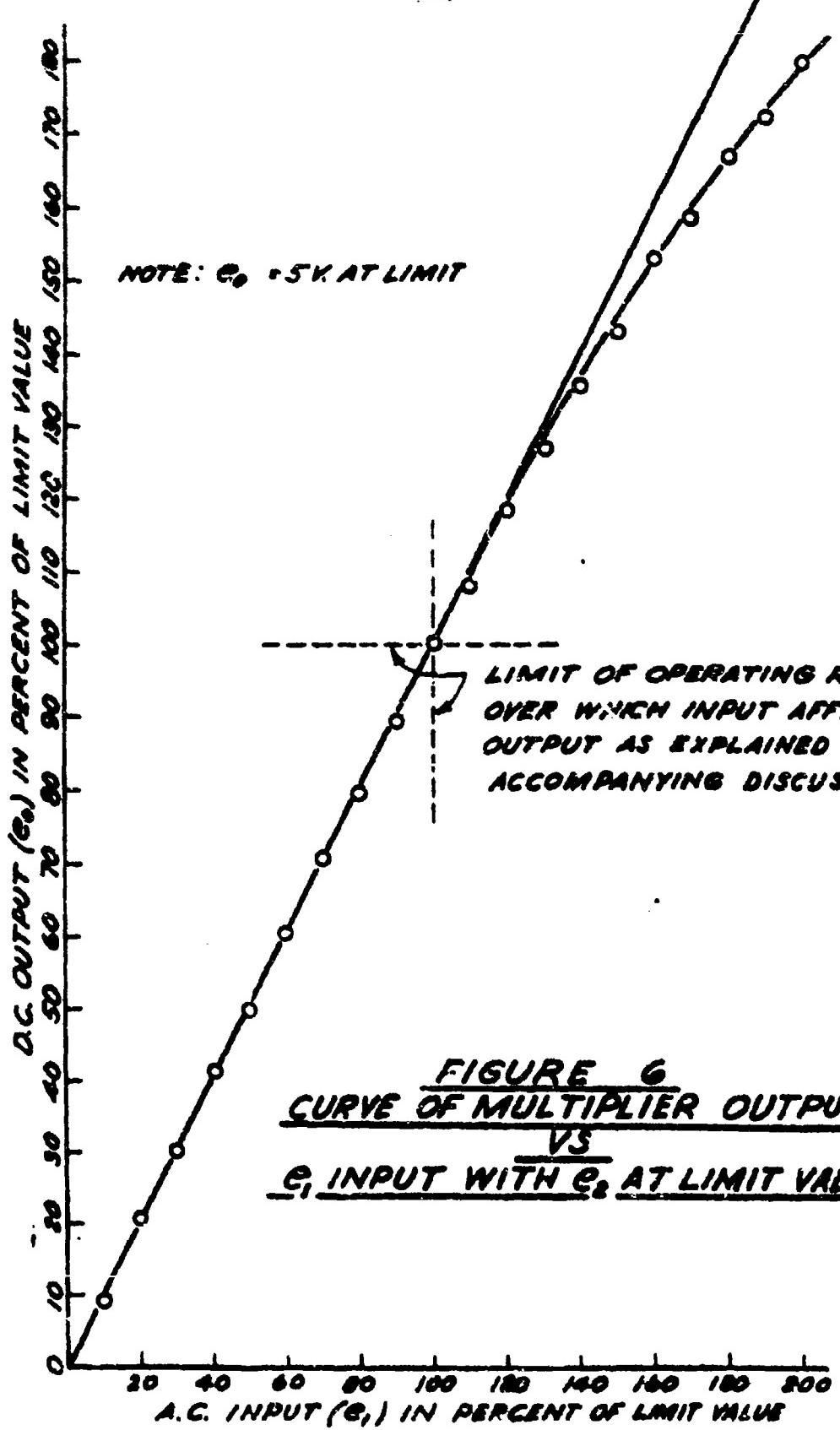
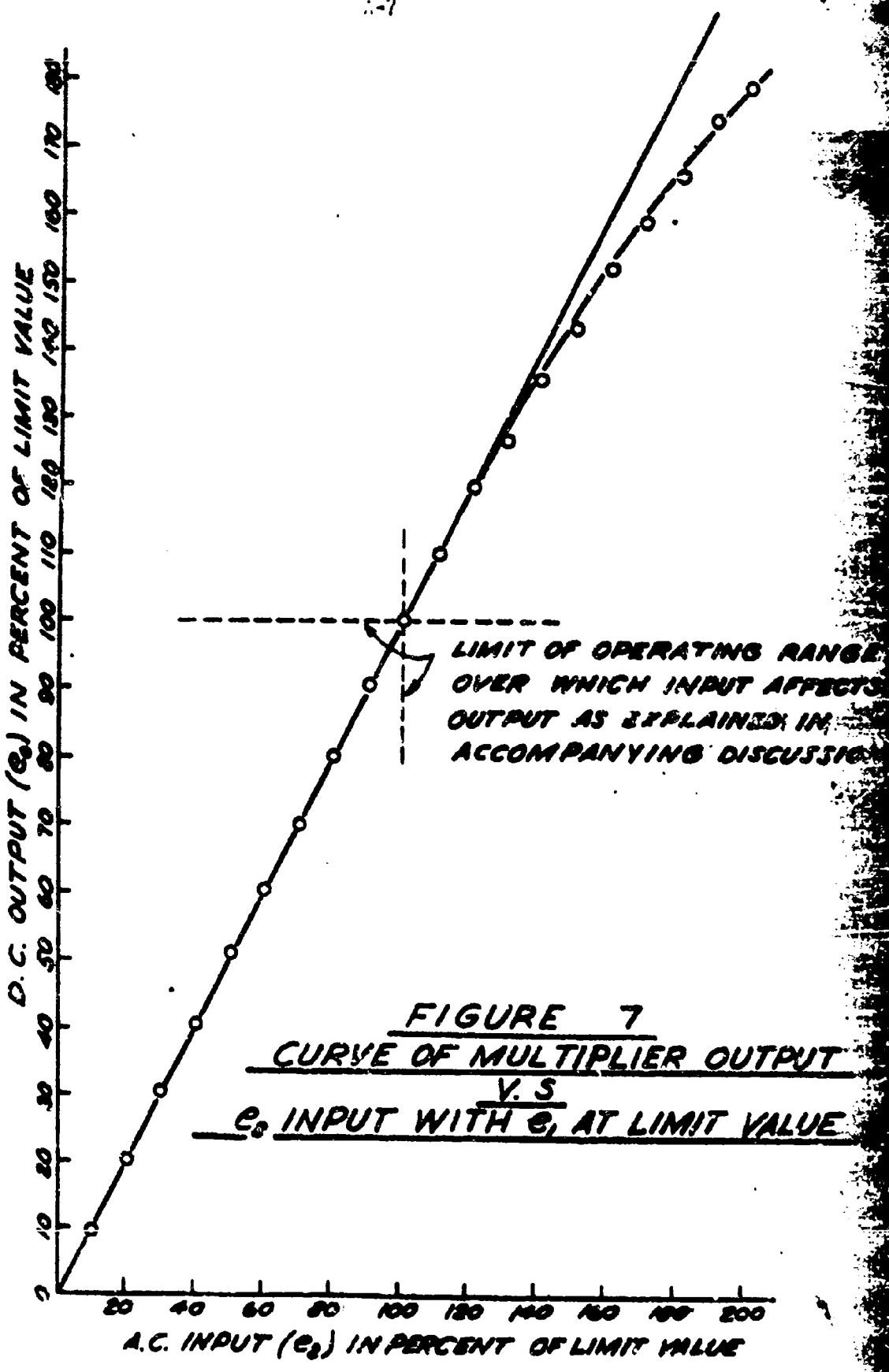


FIGURE 5
SCHEMATIC DIAGRAM OF COMPLETE MULTIPLIER





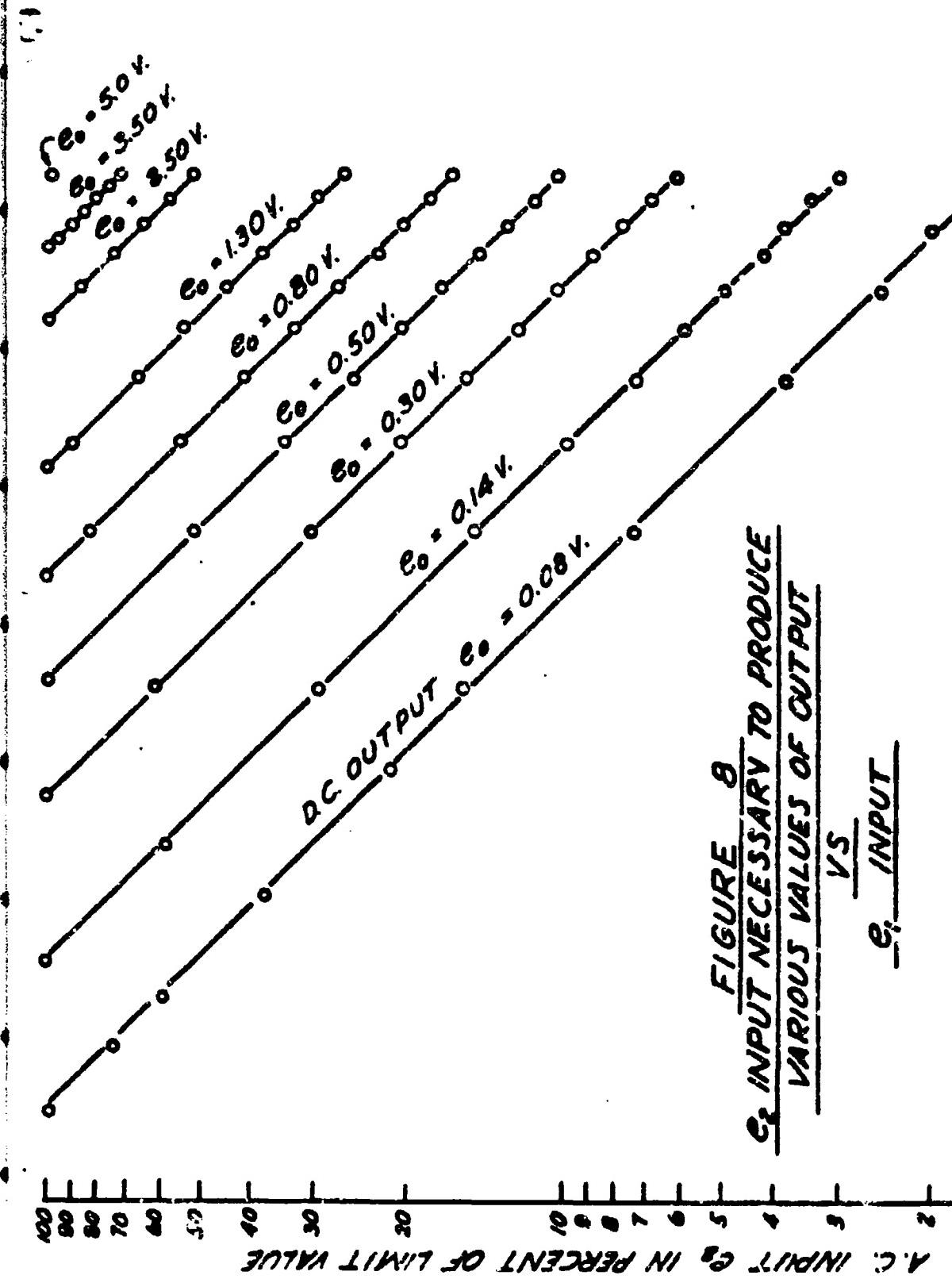


FIGURE
 $\frac{\partial e_2}{\partial e_i}$ INPUT NECESSARY TO PRODUCE
 VARIOUS VALUES OF OUTPUT
 VS
 e_i INPUT

